Lessons from the past and thinking about the future

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Tribute to my Danish Mathematics educator friends

- Ole Skovsmose

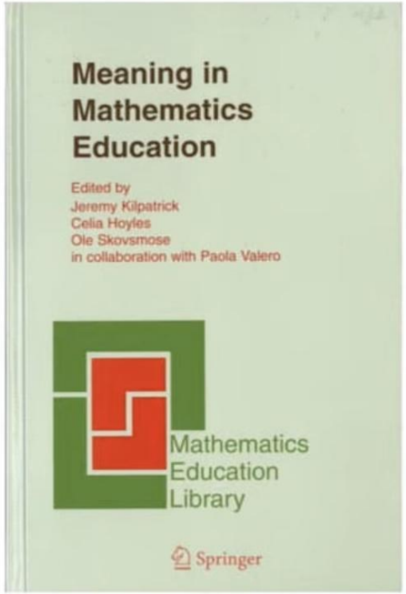
BaCoMET

BaCoMET PROJECT 2. 1986 - 1988

BaCoMET Project 3. 1989 - 1992

BaCoMET Project 4. 1993 - 1996

BaCoMET Project 5. 1996 - 2000



Collaboration with Jeremy Kilpatrick and Ole Skovsmose

Joel Hillel, Anna Sierpinska, Celia Hoyles, Nicolas Balacheff and Colette Laborde

Mogens Niss
Prof Emeritus Roskilde University

1. Learning effective when making an artefact that is **personally** or **socially meaningful**; can be **shared** with others; **reflected upon**, **debugged** (see for example Kafai & Resnick, 1996)
2. Importance of
   - **powerful ideas** embedded in **well-designed constructionist** activity
   - **personal meaning** and **emotional connection…**

ICMI amor

ICMI AMOR project is
https://www.mathunion.org/icmi/projects/icmi-amor

(origin of project https://www.mathunion.org/icmi/awards/amor/about)

Hoyles, C. (2023) Celia Hoyles Unit as winner **Hans Freudenthal Award 2003**

Mathematics Education in the Digital Age: Promise and Reality
https://www.mathunion.org/icmi/celia-hoyles-unit.

structure of my talk

1. Brief reflections on my keynote at ICME, Mexico 2008
2. Cycle of research with digital technologies with examples
3. Potential and challenges for future research & practice
3. The future

the potential of digital technology ICME 2008

dynamic & visual 2D & 3D tools

changing how mathematics

Transforming the mathematical practices of learners and teachers through digital technology

Research in Mathematics Education, Published online: 04 Jul 2018

With response by Paul Drijvers *Freudenthal Institute, Utrecht University*

Stop press: Dialogue about this at BSRLM in Norwich

UK-Netherlands collaboration day Friday 7th Nov the University of East Anglia, prior to BSRLM on Sat 8th Nov

environments more possible

2008 ICME talk: enduring premises……

- centrality of **representation**

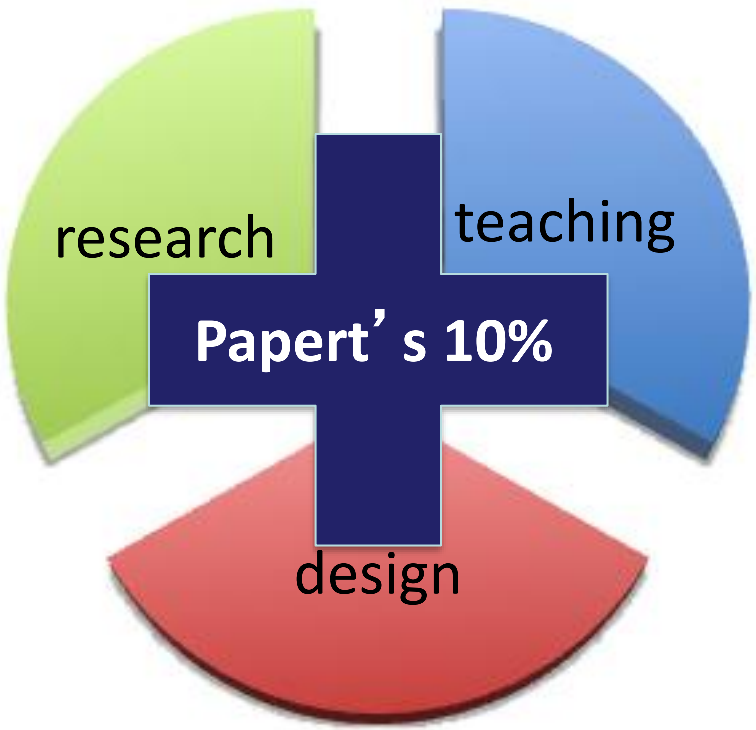
Abstractions are *shaped by* and *expressed in* the medium (Noss & Hoyles, Windows on Mathematical Meanings, 1996)

epistemic agency …the group assumes for the ownership of ideas (Bereiter, 2002; Scardamalia, 2002)
so
less ‘guess what is in the mind of the teacher’ (the didactical paradox, Brousseau)

- seek explanations, produce justifications
- discuss, share, challenge & reflect

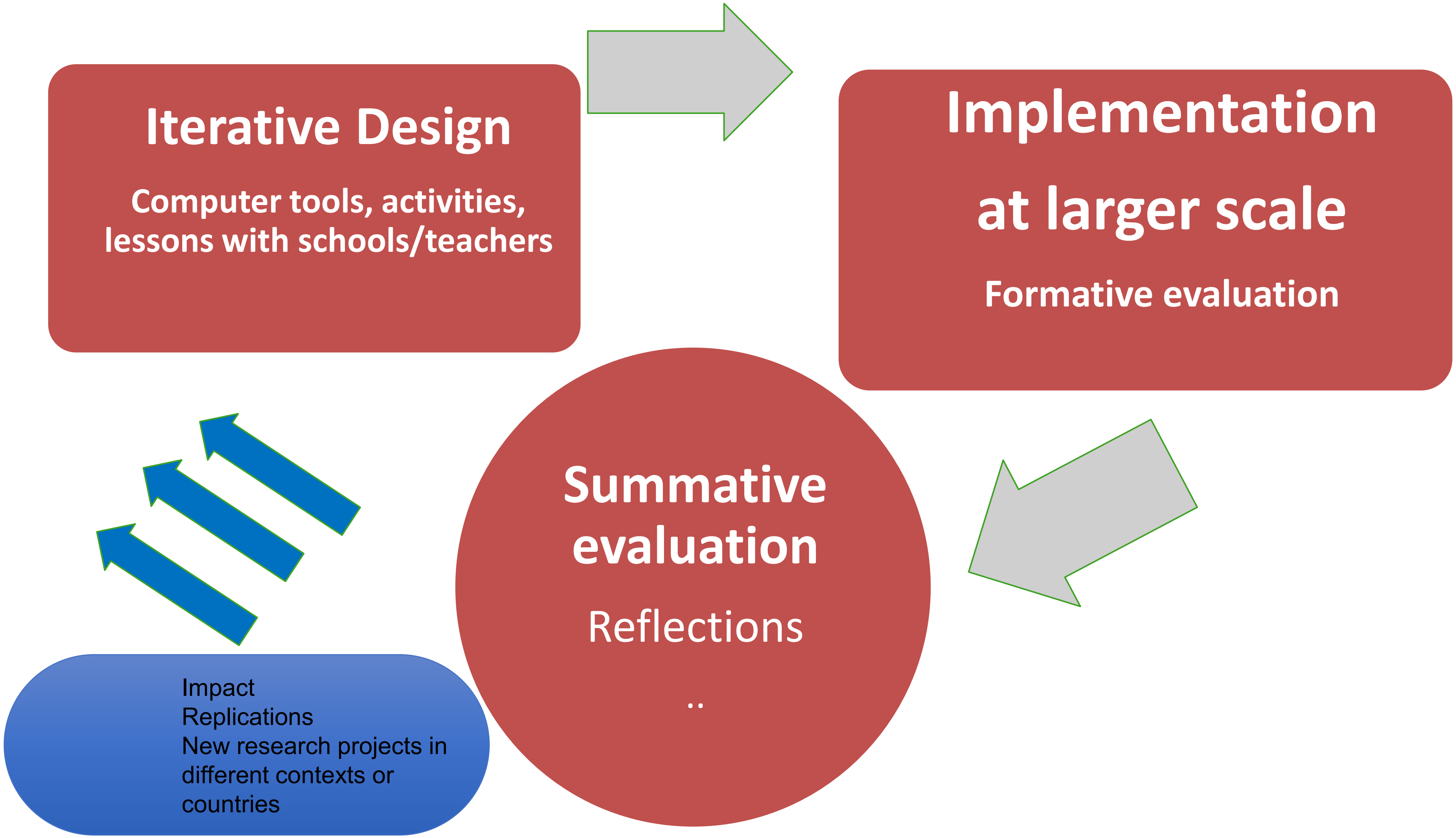
- foster **epistemic agency**

Exploiting digital technologies for 'better' mathematics learning requires ………



ICMI Study: Hoyles. C and Lagrange J-B (eds) (2010) *Mathematics Education and Technology- Rethinking the terrain* Springer

Phases of development of an innovation embedding digital technology

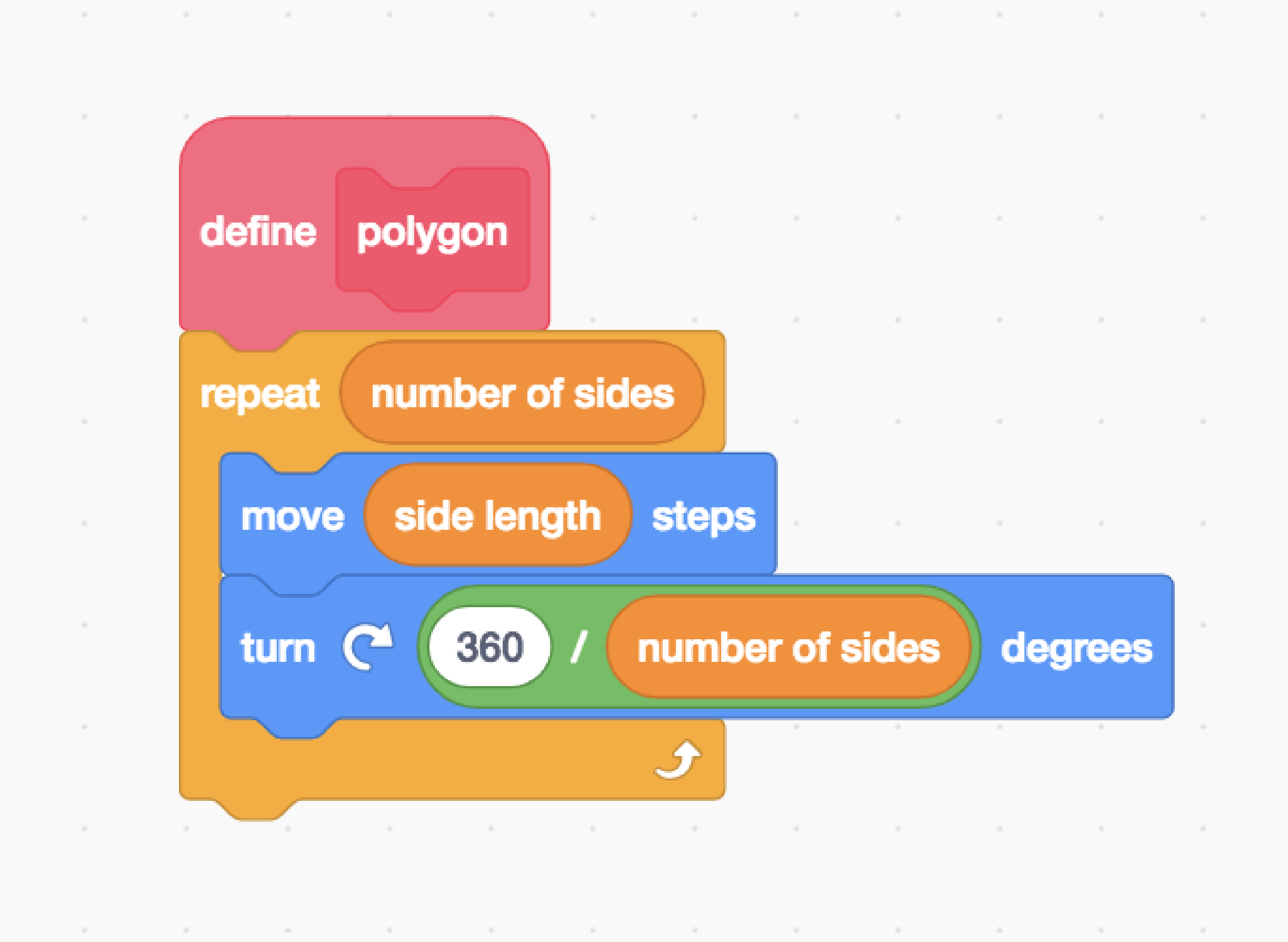


My inspiration…

Reflection on a program can reveal how something is made, its structure …. Important processes of programming..

- build
- trace
- debug
- share…..

A Scratch Program



What we know about programming?..

BUT

....unless learning to program is carefully designed, sequenced and scaffolded, it is the advantaged learners who tend to have most to gain

Key question: What does society need from mathematics and mathematics education?

1. broad base of *quantitative understandings* & *fluency* in mathematical ‘machinery’ not just disconnected facts and procedures
2. **appreciation of the *existence* of mathematical *models* & some grasp of the main *variables & relationships* in these models**
3. being able to *adapt flexibly* to new demands
4. being able *to use tools effectively*, especially digital tools, and *to shape them to new circumstances*
5. being able *to* work *collaboratively*
6. being able *to communicate across boundaries*
7. being able *to reason logically* and *quantitatively* in *different curricula areas* and around *important social issues*

Given all the research in the past with variable outcomes, why should we do better now?

More awareness of

- mathematical models & digital tools especially since COVID
- explicit recognition of critical role of the teacher and need for professional learning
- wider access to web for all… in homes & schools
- new mathematical digital tools & curriculum materials embedding them widely & freely available…

Recognition of danger that

Disadvantage can be made worse by digital divide

Statutory primary National Computing Curriculum 2014 in England for pupils age 6 to 16 years

Key aspect: pupils should **design, build & debug programs**

National Centre for Computing Education NCCE

https://teachcomputing.org

Key question for me was: How did **programming** fit with the mathematics curriculum?

UCL ScratchMaths(SM) project 2014-20……

SM developed a 2-year curriculum with teacher and pupil materials for 9-11year olds in England

- aligned to the **English National Computing** *and* **National Mathematics primary curricula**
- supports the teaching of carefully selected **core ideas of computer programming alongside specific fundamental mathematical concepts**

Aimed to develop computational thinking alongside mathematical thinking

Interdisciplinary team

Led by Professor Dame Celia Hoyles (Mathematics) & Professor Richard Noss (Mathematics) UCL Knowledge LabProfessor Ivan Kalas, (Computing) Comenius University, Bratislava, Slovakia

Dr Laura Benton (Computing) & Piers Saunders, (Mathematics) UCL Knowledge Lab

Prof Dave Pratt (Mathematics) UCL Institute of Education

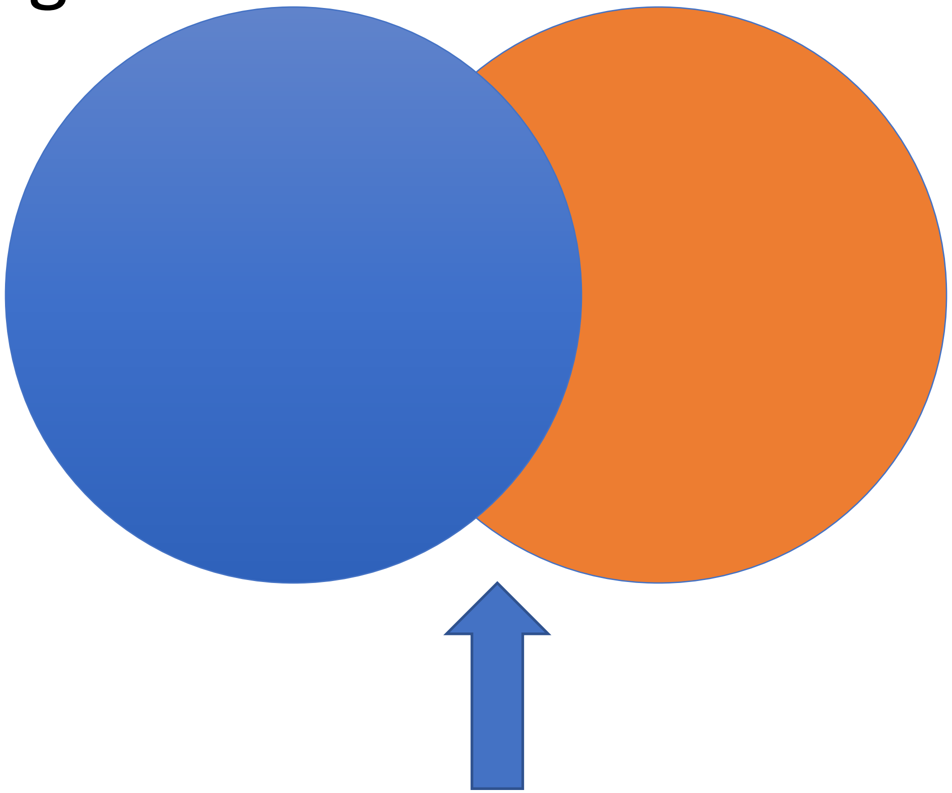
We acknowledge the generous funding of the Education Endowment Foundation

Computational thinking

- seeing a problem and its solution at many levels of detail (abstraction)
- thinking about tasks as a series of logical steps (algorithms)
- understanding that solving a large problem can involve breaking it down into a set of smaller problems (decomposition)
- appreciating that a new problem is likely to be related to other problems the learner has already solved (pattern recognition)
- realising that a solution to a problem can be made in ways that can solve a range of related problems (generalisation)

Computational Thinking alongside Mathematical Thinking

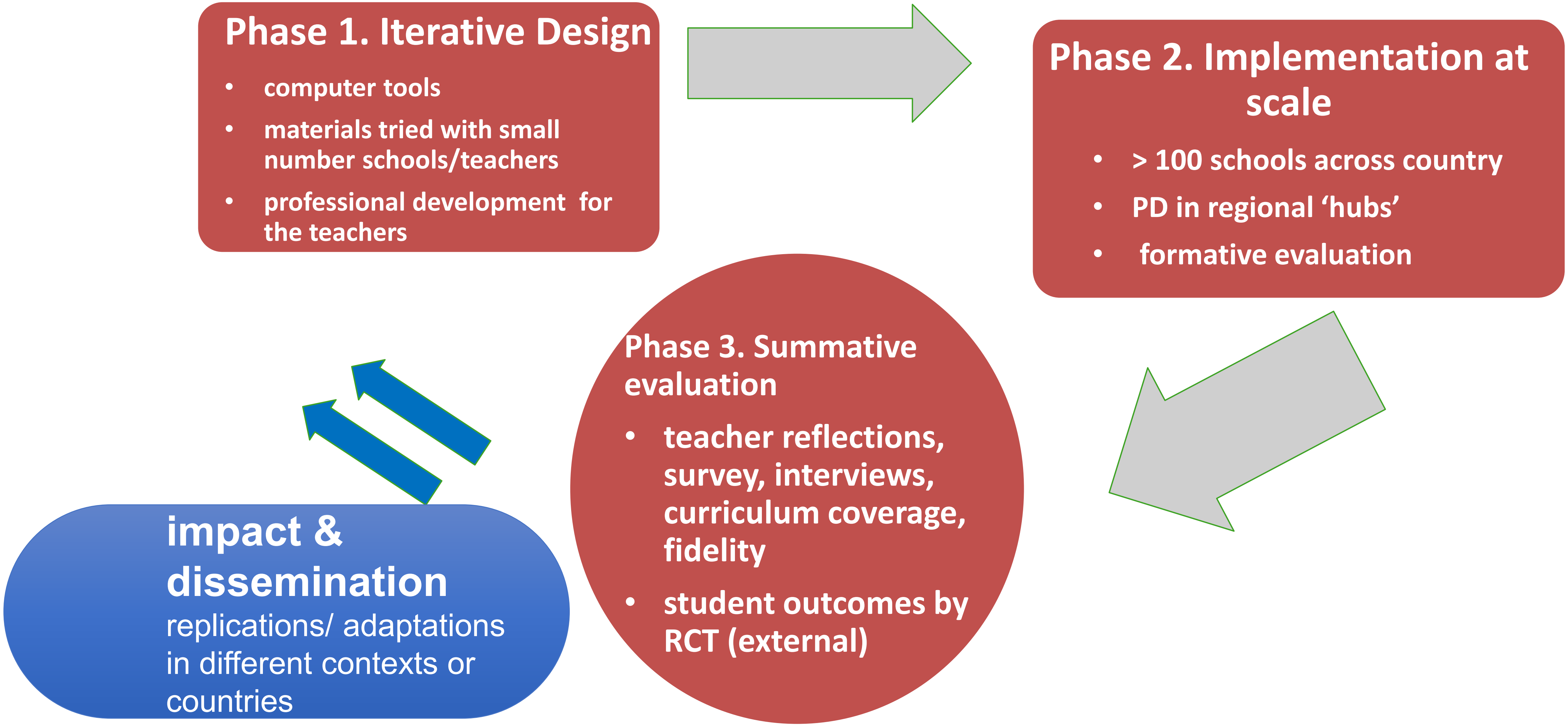
Computational thinking



Mathematical
Thinking

Programming?

Phases of UCL ScratchMaths



A glimpse of the project in action

https://www.ucl.ac.uk/ioe/departments-and-centres/ucl-knowledge-lab/research-and-consultancy/current-research/ucl-scratchmaths

Shift from talking about programming to promoting computational thinking. And now AI?

Reflection on SM project: better assessment ☺

- **Assessment** of student outcomes: mathematics & computing

Need more fine-tuned and nuanced quantitative student outcomes?

- **Assessment** of teacher outcomes
  - understanding of computational concepts & competence & confidence in teaching them
- **Assessment** of **actual** practices in classroom…gender interactions, inclusive practices, fidelity measures

- **Real** commitment to **professional development** for teachers

The Cornerstone Mathematics secondary students age 14

Unit 1 – linear functions

Unit 2 – geometric similarity

Unit 3 – patterns and expressions

A glimpse of the project in action

https://www.youtube.com/watch?v=hvHfAJW7nvI

Lessons for future innovations

Integrate digital technology with **vision** for maths education

Improve **attainment** and **narrow attainment gap**

- gender
- disadvantage

Clear goals for mathematics teaching and learning

- *Content*: focus on ‘hard to teach’ concepts
- *Pedagogy*: explore, engage, excite, envisage, explain, *bridge*
- *Multiple representations* and promote shifts between them
- *Active* and *collaborative learning* promoting dialogue between students
- *Blended learning* on- and off- computer

Professional learning (funded)

- Plan for scaling: regional hubs
- Face- to-face, gap tasks

National constraints

And……

- in England at the end of Year 6 all students take a high-stakes National Mathematics Test, Key Stage 2 test
- Computing is no longer taken so seriously as not part of national assessment
- Moved on to AI

AI and Maths education

- Much to think about together
- Huge potential of AI… we all are using it ☺

BUT some evidence that it can reduce critical thinking

- Tension between using AI as a legitimate learning aid and academic dishonesty (Digital Promise interviews)
- Need coordinated approaches with proper training following iterative design research cycles

BUT a great chance for mathematics *and* computing

"Magic in front of my eyes"

(9 year old student)

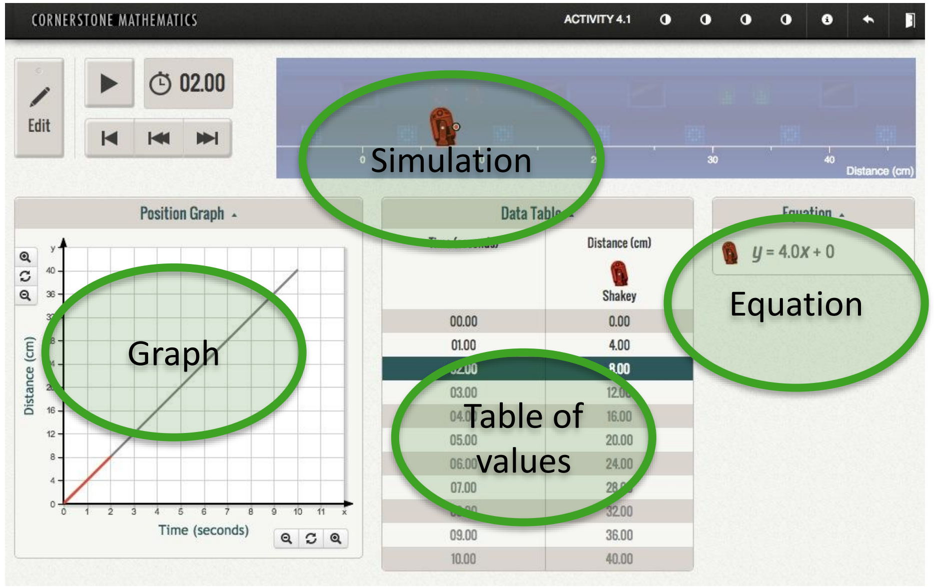
Landmark activity

- disruptive but carefully designed technologies lead to a cognitive breakdown, or a 'situation of non-obviousness'
- the 'aha' moments that show surprise, a rethinking of the mathematics or an extension of previously held ideas
- provides evidence of a developing appreciation of the underlying concept

Designing
Mobile Games
A module on linear functions

Linear functions unit

***Context:** Let's work on a game with robots. We need to set up the mathematics to make our robots move at different speeds.*



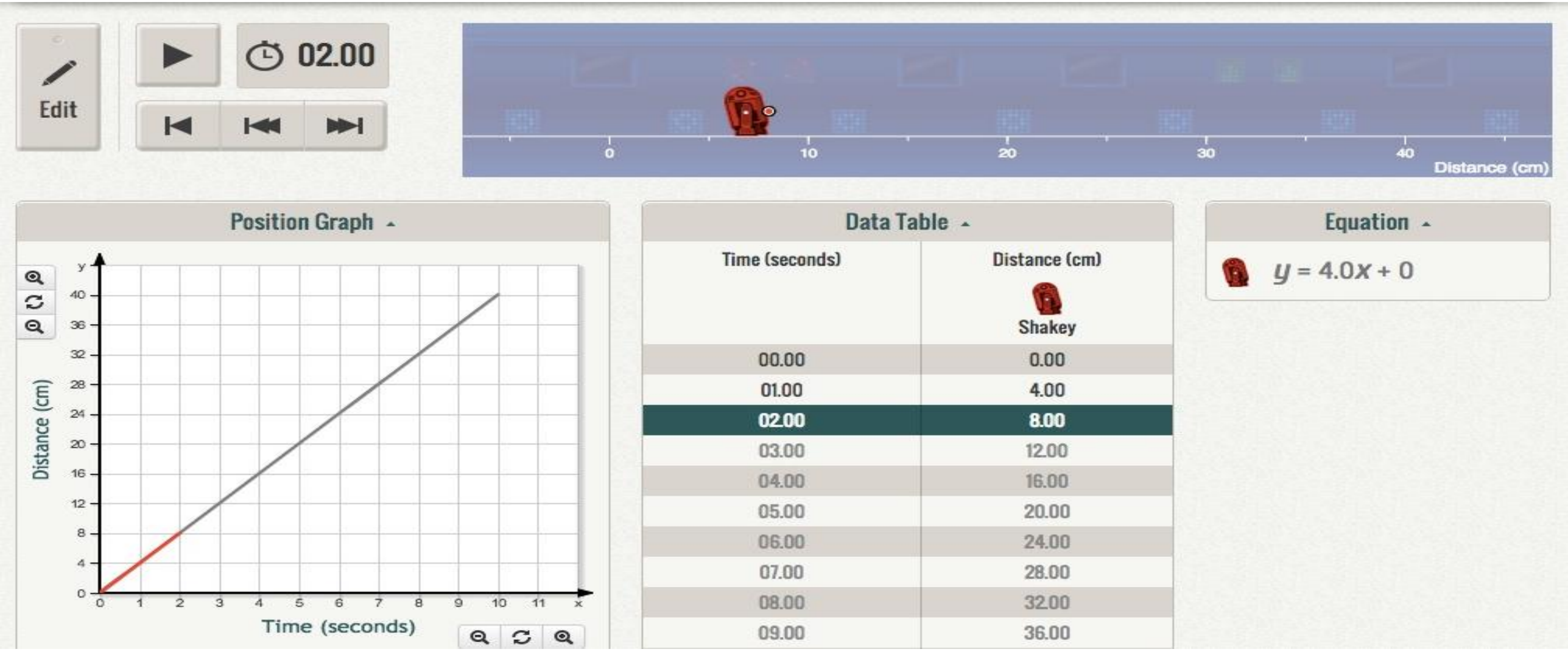
Big mathematical ideas

- **Coordinating** algebraic, graphical, and tabular representations.
- Speed as a context to introduce rates of change.
- y= mx+c as a model of constant velocity motion – the **meaning of m and c in the motion context,**
- **Velocity** as speed with direction and **average velocity.**

Design principles

- dynamic simulation and linking between representations.
- drive the simulation from the graph or the function.
- show/hide representations, as appropriate.

The landmark activity: Shakey the robot



W W W

- Although graphs may look static they can now see that graphs are representations of physical moving objects.
- They learnt how to derive algebra through first understanding graphs
- Learning that a graph tells a story
- Teaching some of the difficult GCSE mathematical concepts through a dynamic approach with early KS3
- MF talks about Y7 final lesson